

## 2.1 – Using Models to Multiply Integers (pp. 64-69)

We can think of multiplication as repeated addition.

5 x3 is the same as adding five 3s: 3 +3 +3 +3 +3

As a sum: 3 +3 +3 +3 +3 =15

As a product: 5 x3 =15

You can also use algebra tiles to model multiplication. (yellow = positive, red = negative)

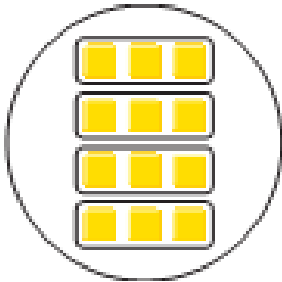
### Ex.1 Multiply: (+4) x(+3)

+4 is a positive integer.

+3 is modelled with 3 yellow tiles

So you have 4 groups of 3, which equals 12 positive tiles

$(+4) \times (+3) = (+3) + (+3) + (+3) + (+3)$   
Make 4 deposits of +3.



### Ex. 2 Multiply (+4) x (-3)

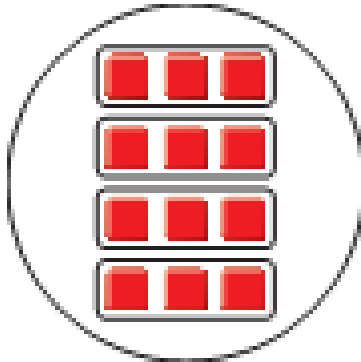
+4 is a positive integer.

-3 is modelled with 3 red tiles.

So you have 4 groups of 3 negatives, which equals 12 negative tiles.

$$(+4) \times (-3) = (-3) + (-3) + (-3) + (-3)$$

Make 4 deposits of  $-3$ .

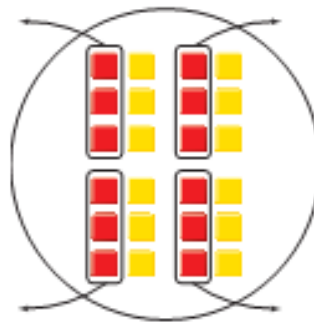


Modelling multiplication of two negative integers with tiles is a little trickier.

**Ex. 3 Multiply:  $(-4) \times (-3)$**

$-4$  is a negative integer.

$-3$  is modelled with 3 red tiles.



So, take 4 sets of 3 red tiles out of the circle.

There are no red tiles to take out.

So, add zero pairs until there are enough red tiles to remove.

Add 12 zero pairs.

Take out 4 sets of 3 red tiles. This leaves 12 yellow (positive tiles), so  $(-4) \times (-3) = +12$

**Ex. 4 Write each multiplication expression as a repeated addition and evaluate.**

a)  $(+7) \times (-4)$

b)  $(+6) \times (+3)$

c)  $(+4) \times (+6)$

d)  $(+5) \times (-6)$

**Ex. 5 Which product does each model represent? Write a multiplication equation for each model.**

a) Deposit 5 sets of 2 red tiles.

b) Deposit 5 sets of 2 yellow tiles.

c) Withdraw 7 sets of 3 red tiles.

d) Withdraw 9 sets of 4 yellow tiles.

e) Deposit 11 sets of 3 yellow tiles.

f) Withdraw 10 sets of 5 red tiles.

## 2.2 – Developing Rules to Multiply Integers (pp. 70-76)

**Multiplying by 0 (Zero property):** any number multiplied by zero equals zero.

$$\text{Ex. } 3 \times 0 = 0; (-10) \times 0 = 0$$

**Multiplying by 1 (Multiplicative Identity):** any number multiplied by 1 equals the original number.

$$\text{Ex. } 23 \times 1 = 23; (-87) \times 1 = (-87)$$

**Commutative Property:** the property of addition and multiplication that states that numbers can be added or multiplied in any order;

$$\text{for example, } 3 + 5 = 5 + 3; 3 \times 5 = 5 \times 3$$

**Distributive Property:** the property stating that a product can be written as a sum or difference of two products; for example,  $a(b + c) = ab + ac$ ,  $a(b - c) = ab - ac$

$$\text{ex. } 3 \times (4 + 5) = 3 \times 4 + 3 \times 5 = 12 + 15 = 27$$

$$\text{or } 3 \times (4 + 5) = 3 \times (9) = 27$$

**If you multiply two integers with the same sign (either two positives or two negatives), the product is positive.**

$$\text{Ex. } (+3) \times (+9) = (+27)$$

$$(-12) \times (-4) = (+48)$$

**If you multiply two integers with opposite signs (one is positive, the other is negative), the product is negative.**

$$\text{Ex. } (-5) \times (+6) = (-30)$$

**Ex. 1 Will each product be positive or negative? How do you know?**

a)  $(-6) \times (+2)$

b)  $(+6) \times (+4)$

c)  $(+4) \times (-2)$

d)  $(-7) \times (-3)$

Ex. 2 Find each product.

a)  $(+8)(-3)$

b)  $(-5)(-4)$

c)  $(-3)(+9)$

d)  $(+7)(-6)$

e)  $(+10)(-3)$

f)  $(-7)(-6)$

g)  $(0)(-8)$

h)  $(+10)(-1)$

i)  $(-7)(-8)$

j)  $(+9)(-9)$

Ex. 3 Gaston withdrew \$26 from his bank account each week for 17 weeks. Use integers to find the total amount Gaston withdrew over the 17 weeks. Show your work

Ex. 4 Amelie was doing a math question. The answer she got did not match the answer in the answer key. So, she asked a friend to look at her work.

$$\begin{aligned} (+60) \times (-18) \\ &= (+60) \times [(-20) + (+2)] \\ &= [(+60) \times (-20)] + [(+60) \times (+2)] \\ &= (+1200) + (+120) \\ &= +1320 \end{aligned}$$

a) What was Amelie's error? b) Correct Amelie's error. What is the correct answer?

## 2.3 – Using Models to Divide Integers (pp. 77-83)

**Ex. 1** The 1850-km Iditarod dogsled race lasts from 10 to 17 days. One night, the temperature fell  $2^{\circ}\text{C}$  each hour for a total change of  $-12^{\circ}\text{C}$ . Use integers to find how many hours this change in temperature took.

$-2$  represents a fall of  $2^{\circ}\text{C}$ .

$-12$  represents a change of  $-12^{\circ}\text{C}$ .

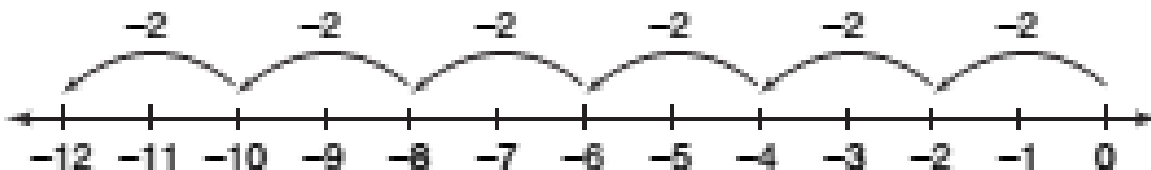
Using integers, we need to find how many  $-2$ s take us to  $-12$ ;

that is,  $(-12) \div (-2)$ .

Start at 0.

Move 2 units left.

Continue to move 2 units left until you reach  $-12$ .



Six moves of 2 units left were made.

So,  $(-12) \div (-2) = (+6)$

**Ex. 2** Write a related multiplication equation for each division equation.

a)  $(+25) \div (+5) = +5$

b)  $(+24) \div (-2) = -12$

c)  $(-14) \div (-7) = +2$

d)  $(-18) \div (+6) = -3$

**Ex. 3** The temperature fell  $4^{\circ}\text{C}$  each hour for a total change of  $-20^{\circ}\text{C}$ . Use integers to find the number of hours the change in temperature took.

**Ex. 4** Abraham used the Internet to find the low temperature in six Western Canadian cities on a particular day in January. He recorded the temperatures in a table

City	Low Temperature ( $^{\circ}\text{C}$ )
Whitehorse	-10
Iqaluit	-7
Vancouver	+9
Edmonton	-1
Winnipeg	-9
Saskatoon	-6

- Find the mean low temperature for these cities on that day.
- The low temperature in Regina for the same day was added to the table. The mean low temperature for the seven cities was  $-3^{\circ}\text{C}$ . What was the temperature in Regina?

## 2.4 – Developing Rules to Divide Integers (pp. 84- 89)

**Dividing by zero is impossible (the divisor can never be zero)**

Ex.  $3 \div 0$  is not possible (but  $0 \div 3 = 0$ )

**Dividing a number by one gives you the same number.**

Ex.  $23 \div 1 = 23$

Division can be represented with a division sign ( $\div$ ) or as a fraction

Ex.  $6 \div 3 = 6/3 = \frac{6}{3}$

**If you divide two integers with the same sign (either two positives or two negatives), the quotient is positive.**

Ex.  $(+8) \div (+2) = (+4)$        $(-27) \div (-3) = (+9)$

**If you divide two integers with opposite signs (one positive and one negative), the quotient is negative**

Ex.  $(-45) \div (+5) = (-9)$        $(+64) \div (-8) = (-8)$

**Ex. 1 Shannon made withdrawals of \$14 from her bank account. She withdrew a total of \$98. Use integers to find how many withdrawals Shannon made.**

–14 represents a withdrawal of \$14.

–98 represents a total withdrawal of \$98.

Divide to find the number of withdrawals.

Since each integer has 2 digits, use a calculator.

$(-98) \div (-14) = +7$

Shannon made 7 withdrawals of \$14.



**Ex. 2 Will each quotient be positive or negative? How do you know?**

a)  $(-45) \div (+5)$

b)  $(+16) \div (+8)$

c)  $(+24) \div (-2)$

d)  $(-30) \div (-6)$

**Ex. 3 Find each quotient.**

a)  $(+12) \div (+4)$

b)  $(-15) \div (-3)$

c)  $(-18) \div (+9)$

d)  $(+81) \div (-9)$

e)  $(+72) \div (-8)$

f)  $(-64) \div (-8)$

g)  $(-14) \div (+1)$

h)  $(+54) \div (-6)$

i)  $(-27) \div (-3)$

j)  $(+32) \div (+4)$

**Ex. 4 Winnie used the money in her savings account to pay back a loan from her mother. Winnie paid back her mother in 12 equal weekly payments. Over the 12 weeks, the balance in Winnie's savings account decreased by \$132. By how much did her balance change each week?**

## 2.5 – Order of Operations with Integers (pp. 90-95)

Recall the order of operations with whole numbers and integers.

- Do the operations in brackets first.
- Multiply and divide, in order, from left to right.
- Add and subtract, in order, from left to right.

### Example 1

Evaluate:  $[(-6) + (-2)] \div (-4) + (-5)$

#### ▶ A Solution

$$\begin{aligned} & [(-6) + (-2)] \div (-4) + (-5) && \text{Do the operation in square brackets first.} \\ & \underbrace{(-6) + (-2)}_{-8} \div (-4) + (-5) \\ & = (-8) \div (-4) + (-5) && \text{Divide.} \\ & = (+2) + (-5) && \text{Add.} \\ & = -3 \end{aligned}$$

### Example 2

Evaluate:  $\frac{2 + 4 \times (-8)}{-6}$

#### ▶ A Solution

$$\begin{aligned} & \frac{2 + 4 \times (-8)}{-6} && \text{Evaluate the numerator.} \\ & && \text{Multiply.} \\ & = \frac{2 + (-32)}{-6} && \text{Add.} \\ & = \frac{-30}{-6} && \text{Divide.} \\ & = 5 \end{aligned}$$

If an integer does not have a sign, it is assumed to be positive; for example,  $2 = +2$ . Then we do not need to put the number in brackets.

Ex. 3 State which operation you do first.

a)  $7 + (-1) \times (-3)$

b)  $(-18) \div (-6) - (-4)$

c)  $6 + (-4) - (-2)$

d)  $(-2)[7 + (-5)]$

e)  $(-3) \times (-4) \div (-1)$

f)  $8 - 3 + (-4) \div (-1)$

Ex. 4

Evaluate. Show all steps.

a)  $\frac{(-7) \times 4 + 8}{4}$

b)  $\frac{4 + (-36) \div 4}{-5}$

c)  $\frac{-32}{(-6)(-2) - (-4)}$

d)  $\frac{9}{(-3) + (-18) \div 3}$

